Influence of external magnetic field on dynamics of open quantum systems

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The influence of an external magnetic field on the non-Markovian dynamics of an open two-dimensional quantum system is investigated. The fluctuations of collective coordinate and momentum and transport coefficients are studied for a charged harmonic oscillator linearly coupled to a neutral bosonic heat bath. It is shown that the dissipation of collective energy slows down with increasing strength of the external magnetic field. The role of magnetic field in the diffusion processes is illustrated by several examples.

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I. INTRODUCTION

The problem of transport in a two-dimensional quantum system under the influence of an external magnetic field and dissipation to its environment is of great interest in the fields of atomic, nuclear, and plasma physics, astrophysics, and quantum information theory in condensed matter systems. The theory of open quantum systems, developed in great part due to the field of quantum optics, is a central component of the modern theory of quantum measurement, studies on quantum chaos, condensed matter physics, and precision metrology [1]. In atomic physics, much attention is focused on the hydrogen atom in a magnetic field for which the experimental and theoretical studies yield excellent insights into semiclassical and quantum aspects of nonintegrable systems (see, for example, Ref. [2]). In nuclear physics, the study of nuclear properties in the strong field of a magnetic trap seems to be attractive. The observation scheme of the simultaneous violation of parity and time-reversal invariance is based on the measurement of the linear polarization of the γ transitions produced by the deexcitation of isomeric states of nuclei in the magnetic field at low temperature [3]. The intensive investigations deal with the influence of an external magnetic field on such systems as quantum dots, quantum wires, and two-dimensional electronic systems [4]. The characteristics of plasma in a homogeneous external field is of interest in the physics of gas discharge [6]. Note that in the present laboratory conditions one can obtain magnetic fields up to 10^7 G.

The influence of the magnetic field on the properties of the quantum system was investigated with different approaches. Using the phenomenological Markovian Fokker-Planck equation for the Wigner probability function, the problem of a quantum description of a damped isotropic two-dimensional harmonic oscillator in a uniform magnetic field was studied in Ref. [7] in the case of arbitrary relations between the proper oscillator frequency, damping coefficients, and temperature. The relations between the phenomenological diffusion coefficients ensuring the positivity of the reduced density matrix at each moment of time were obtained in Ref. [7]. The problem of a quantum system coupled to a quantum mechanical heat bath can be formulated in terms of the quantum Langevin equation. An early derivation of the one-dimensional quantum Langevin equation with external force was performed in Ref. [8]. It was shown that the particle coupled to a heat bath and influenced by an arbitrary force to the fixed center exhibits Brownian motion. As an application of the quantum Markovian Langevin equation the dynamics of a quantum system coupled to a heat bath in the presence of a one-dimensional harmonic oscillator potential was considered. The authors of Ref. [8] established rigorous conditions for instantaneous dissipation. Including the magnetic field in the quantum non-Markovian Langevin equation, the effects of dissipation and magnetic field on the localization of charged particles were investigated in Refs. [9,10]. It was found that weak dissipation delocalizes the oscillation of a charged particle when the magnetic field is stronger than a certain critical value [10]. A charged particle moving in a confined parabolic potential and magnetic field was treated in Ref. [11] in the quantum dissipative regime. It should be noted that the time-dependent friction and diffusion coefficients were not analytically derived and numerically studied in the above-mentioned references.

The aim of the present work is the development and analysis of the quantum Langevin equation treatment of damped transport in a magnetic field beyond the Markov approximation (instantaneous dissipation, δ-correlated fluctuations) and the weak-coupling limit. We make a two-dimensional generalization of the Langevin formalism which has been developed for non-Markovian noise in Refs. [12,13]. Our formalism is valid at arbitrary coupling strengthes and hence at arbitrary low temperatures. We obtain and solve the two-dimensional quantum non-Markovian Langevin equations to investigate the transport properties and energy dissipation of an open quantum system in the presence of a uniform external magnetic field and linear coupling in the coordinate between the collective harmonic oscillator and bosonic heat bath. The influence of external axisymmetric magnetic field on the friction and diffusion coefficients of the two-dimensional quantum system is studied. We look also for the fluctuations of collective variables. By considering the fluctuations of collective coordinates, we estimate the role of magnetic field in the channeling of the particle in a crystal.

In Sec. II we will derive and solve the quantum two-dimensional non-Markovian Langevin equations with an external magnetic field. In Sec. III the transport coefficients will be obtained by considering first and second moments of the stochastic dissipative equations. A discussion and illustrative numerical results will be presented in Sec. IV.

II. NON-MARKOVIAN LANGEVIN EQUATIONS WITH EXTERNAL MAGNETIC FIELD

In order to investigate the influence of external fields on the dynamics of open quantum systems, we consider the motion of a collective charged subsystem in a two-dimensional parabolic potential (in xy plane) surrounded by a neutral bosonic heat bath in the presence of a perpendicular axisymmetric magnetic field (along the z axis). In the case of linear coupling in coordinates between this subsystem and heat bath the total Hamiltonian of the collective subsystem+heat bath is as follows:

$$H=\frac{1}{2\mu}[\mathbf{p}-e\mathbf{A}(x,y)]^2+\frac{\mu}{2}(\omega_x^2x^2+\omega_y^2y^2)+\sum_\nu \hbar\omega_\nu b_\nu^\dagger b_\nu + \sum_\nu (x\alpha_\nu+yg_\nu)(b_\nu^\dagger+b_\nu), \quad (1)$$

where $\mathbf{A}=\left(-\frac{1}{2}yB,\frac{1}{2}xB,0\right)$ is the vector potential of the magnetic field with strength $B=|\mathbf{B}|$, $\mathbf{p}$ is the canonically conjugated momentum, ω_x and ω_y are the collective frequencies, and b_ν^+ and b_ν are the phonon creation and annihilation operators of the heat bath. The coupling parameters α_ν and g_ν are determined as in Ref. [12]:

$$\alpha_\nu^2=\frac{2\mu\omega_x\lambda_x^0}{\hbar}G_\nu^2,$$

$$g_\nu^2=\frac{2\mu\omega_y\lambda_y^0}{\hbar}G_\nu^2, \quad (2)$$

where $\lambda_{x,y}^0$ are parameters which measure the average strengths of the interactions and G_ν are the coupling constants. In Eq. (1) the first term includes the magnetic field energy and the last term describes the interaction between the collective subsystem and heat bath. The bosonic heat bath is modeled by an ensemble of noninteracting harmonic oscillators with frequencies ω_ν. The coupling between the heat bath and collective subsystem is linear in coordinates. The coupling term and the external magnetic field do not affect each other.

For convenience, we introduce new definitions for the momentum:

$$\pi_x=p_x+\frac{1}{2}\mu\omega_L y,$$

$$\pi_y=p_y-\frac{1}{2}\mu\omega_L x, \quad (3)$$

where $\omega_L=eB/\mu$ is the cyclotron frequency and $[\pi_x,\pi_y]=-[\pi_y,\pi_x]=\imath\hbar\mu\omega_L$. Therefore, the total Hamiltonian (1) is transformed into the form

$$H=\frac{1}{2\mu}(\pi_x^2+\pi_y^2)+\frac{\mu}{2}(\omega_x^2x^2+\omega_y^2y^2)+\sum_\nu \hbar\omega_\nu b_\nu^\dagger b_\nu+\sum_\nu (x\alpha_\nu+yg_\nu)(b_\nu^\dagger+b_\nu). \quad (4)$$

The system of Heisenberg equations for the operators x, y, π_x, and π_y, and the bath phonon operators b_ν and b_ν^+ is obtained by commuting them with H:

$$\dot{x}(t)=\frac{i}{\hbar}[H,x]=\frac{\pi_x(t)}{\mu},$$

$$\dot{y}(t)=\frac{i}{\hbar}[H,y]=\frac{\pi_y(t)}{\mu},$$

$$\dot{\pi}_x(t)=\frac{i}{\hbar}[H,\pi_x]=\pi_y(t)\omega_L-\mu\omega_x^2x(t)-\sum_\nu \alpha_\nu(b_\nu^\dagger+b_\nu),$$

$$\dot{\pi}_y(t)=\frac{i}{\hbar}[H,\pi_y]=-\pi_x(t)\omega_L-\mu\omega_y^2y(t)-\sum_\nu g_\nu(b_\nu^\dagger+b_\nu) \quad (5)$$

and

$$\dot{b}_\nu^\dagger(t)=\frac{i}{\hbar}[H,b_\nu^\dagger]=i\omega_\nu b_\nu^\dagger(t)+\frac{i}{\hbar}[\alpha_\nu x(t)+g_\nu y(t)],$$

$$\dot{b}_\nu(t)=\frac{i}{\hbar}[H,b_\nu]=-i\omega_\nu b_\nu(t)-\frac{i}{\hbar}[\alpha_\nu x(t)+g_\nu y(t)]. \quad (6)$$

The solutions of Eqs. (6) are

$$b_\nu^\dagger(t)=f_\nu^\dagger(t)-\frac{\alpha_\nu x(t)+g_\nu y(t)}{\hbar\omega_\nu}+\frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau\dot{x}(\tau)e^{i\omega_\nu(t-\tau)}+\frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau\dot{y}(\tau)e^{i\omega_\nu(t-\tau)},$$

$$b_\nu(t)=f_\nu(t)-\frac{\alpha_\nu x(t)+g_\nu y(t)}{\hbar\omega_\nu}+\frac{\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau\dot{x}(\tau)e^{-i\omega_\nu(t-\tau)}+\frac{g_\nu}{\hbar\omega_\nu}\int_0^t d\tau\dot{y}(\tau)e^{-i\omega_\nu(t-\tau)}, \quad (7)$$

where

$$f_\nu(t)=\left(b_\nu(0)+\frac{i}{\omega_\nu}B_\nu(0)\right)e^{-i\omega_\nu t},$$

$$B_\nu(t)=\alpha_\nu x(t)+g_\nu y(t).$$

Therefore,

$$b_\nu^\dagger(t) + b_\nu(t) = f_\nu^\dagger(t) + f_\nu(t) - 2\frac{\alpha_\nu x(t) + g_\nu y(t)}{\hbar\omega_\nu} + \frac{2\alpha_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{x}(\tau)\cos[\omega_\nu(t-\tau)] + \frac{2g_\nu}{\hbar\omega_\nu}\int_0^t d\tau \dot{y}(\tau)\cos[\omega_\nu(t-\tau)]. \quad (8)$$

Substituting Eq. (8) into Eqs. (5), we eliminate the bath variables from the equations of motion of the collective subsystem and obtain the nonlinear integro-differential stochastic dissipative equations

$$\dot{x}(t) = \frac{\pi_x(t)}{\mu},$$

$$\dot{y}(t) = \frac{\pi_y(t)}{\mu},$$

$$\dot{\pi}_x(t) = \pi_y(t)\omega_L - x(t)\mu\omega_x^2\left(1 - \frac{1}{\omega_x^2}\sum_\nu \frac{2\alpha_\nu^2}{\mu\hbar\omega_\nu}\right) - \frac{1}{\mu}\int_0^t d\tau K_\alpha(t,\tau)\pi_x(\tau) - F_\alpha(t),$$

$$\dot{\pi}_y(t) = -\pi_x(t)\omega_L - y(t)\mu\omega_y^2\left(1 - \frac{1}{\omega_y^2}\sum_\nu \frac{2g_\nu^2}{\mu\hbar\omega_\nu}\right) - \frac{1}{\mu}\int_0^t d\tau K_g(t,\tau)\pi_y(\tau) - F_g(t). \quad (9)$$

The presence of the integral parts in these equations indicates the non-Markovian character of the system. Since in comparison with Refs. [14,15] we do not introduce the counterterm in the Hamiltonian, the stiffnesses of the potentials are renormalized in the equations above. Due to the operators

$$F_\alpha(t) = \sum_\nu F_\alpha^\nu(t) = \sum_\nu \alpha_\nu(f_\nu^\dagger + f_\nu),$$

$$F_g(t) = \sum_\nu F_g^\nu(t) = \sum_\nu g_\nu(f_\nu^\dagger + f_\nu),$$

which play the role of random forces in the coordinates, Eqs. (9) can be called the generalized nonlinear quantum Langevin equations. Following the usual procedure of statistical mechanics, we identify these operators as fluctuations because of the uncertainty in the initial conditions for the bath operators. To specify the statistical properties of the fluctuations, we consider an ensemble of initial states in which the fluctuations have a Gaussian distribution with zero average value

$$\langle\langle F_\alpha^\nu(t)\rangle\rangle = \langle\langle F_g^\nu(t)\rangle\rangle = 0. \quad (10)$$

Here, the symbol $\langle\langle\cdots\rangle\rangle$ denotes the average over the bath. We assume that there are no correlations between $F_\alpha^\nu(t)$ and $F_g^\nu(t)$, so that

$$\sum_\nu \frac{\alpha_\nu g_\nu}{\hbar\omega_\nu} \equiv 0. \quad (11)$$

The dissipative kernels in Eqs. (9) are

$$K_\alpha(t-\tau) = \sum_\nu \frac{2\alpha_\nu^2}{\hbar\omega_\nu}\cos(\omega_\nu[t-\tau]),$$

$$K_g(t-\tau) = \sum_\nu \frac{2g_\nu^2}{\hbar\omega_\nu}\cos(\omega_\nu[t-\tau]). \quad (12)$$

Since these kernels do not contain the phonon occupation numbers, they are independent of the temperature T of the heat bath. The temperature enters in the analysis through the specification of the distribution of the initial conditions. We use Bose-Einstein statistics for the heat bath:

$$\langle\langle f_\nu^\dagger(t) f_{\nu'}^{+}(t')\rangle\rangle = \langle\langle f_\nu(t) f_{\nu'}(t')\rangle\rangle = 0,$$

$$\langle\langle f_\nu^\dagger(t) f_{\nu'}(t')\rangle\rangle = \delta_{\nu,\nu'} n_\nu e^{i\omega_\nu(t-t')},$$

$$\langle\langle f_\nu(t) f_{\nu'}^\dagger(t')\rangle\rangle = \delta_{\nu,\nu'}(n_\nu+1)e^{-i\omega_\nu(t-t')}, \quad (13)$$

with occupation numbers for phonons $n_\nu = [\exp(\hbar\omega_\nu/T) - 1]^{-1}$ depending on T. Using the properties of random forces, we obtain the quantum fluctuation-dissipation relations

$$\sum_\nu \varphi_{\alpha\alpha}^\nu(t,t')\frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} = K_\alpha(t-t'),$$

$$\sum_\nu \varphi_{gg}^\nu(t,t')\frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} = K_g(t-t'),$$

where

$$\varphi_{kk}^\nu(t,t') = 2k_\nu^2[2n_\nu + 1]\cos(\omega_\nu[t-t'])$$

are the symmetrized correlation functions $\varphi_{kk}^\nu(t,t') = \langle\langle F_k^\nu(t)F_k^\nu(t') + F_k^\nu(t')F_k^\nu(t)\rangle\rangle$, $k=\alpha,g$. The quantum fluctuation-dissipation relations differ from the classical ones and are reduced to them in the limit of high temperature T (or $\hbar\to 0$): $\Sigma_\nu\varphi_{\alpha\alpha}^\nu(t,t') = 2TK_\alpha(t-t')$ and $\Sigma_\nu\varphi_{gg}^\nu(t,t') = 2TK_g(t-t')$.

It is convenient to introduce the spectral density $D(\omega)$ of the heat bath excitations which allows us to replace the sum over different oscillators ν by the integral over the frequency: $\Sigma_\nu\cdots \to \int_0^\infty d\omega D(\omega)\cdots$. This replacement is accompanied by the following replacements: $G_\nu \to G_\omega$, $\omega_\nu \to \omega$, $n_\nu \to n_\omega$. Let us consider the following spectral functions [15–18]

$$D(\omega)\frac{|G(\omega)|^2}{\hbar^2\omega} = \frac{1}{\pi}\frac{\gamma^2}{\gamma^2+\omega^2}, \quad (14)$$

where the memory time γ^{-1} of the dissipation is inverse to the phonon bandwidth of the heat bath excitations which are coupled with the collective oscillator. This is Ohmic dissipation with a Lorenzian cutoff (Drude dissipation). The relaxation time of the heat bath should be much less than the period of the collective oscillator—i.e., $\gamma \gg \omega_{x,y}$. If we rewrite the sum Σ_ν as the integral over the bath frequencies with the density of states, we obtain

$$K_\alpha(t) = \kappa_x^2 \frac{\lambda_x^0 \gamma}{\hbar} e^{-\gamma|t|},$$

$$K_g(t) = \kappa_y^2 \frac{\lambda_y^0 \gamma}{\hbar} e^{-\gamma|t|},$$

where $\kappa_x^2 = 2\mu\omega_x\hbar$ and $\kappa_y^2 = 2\mu\omega_y\hbar$.

As in Ref. [12], the system of equations (9) is solved by applying the Laplace transformations. Here, we do not present the tedious algebra and bring only the solution of this system of equations:

$$x(t) = A_1(t)x(0) + A_2(t)y(0) + A_3(t)\pi_x(0) + A_4(t)\pi_y(0) - I_x(t) - I'_x(t),$$

$$y(t) = B_1(t)x(0) + B_2(t)y(0) + B_3(t)\pi_x(0) + B_4(t)\pi_y(0) - I_y(t) - I'_y(t),$$

$$\pi_x(t) = C_1(t)x(0) + C_2(t)y(0) + C_3(t)\pi_x(0) + C_4(t)\pi_y(0) - I_{\pi_x}(t) - I'_{\pi_x}(t),$$

$$\pi_y(t) = D_1(t)x(0) + D_2(t)y(0) + D_3(t)\pi_x(0) + D_4(t)\pi_y(0) - I_{\pi_y}(t) - I'_{\pi_y}(t), \quad (15)$$

where $I_x(t) = \int_0^t A_3(\tau)F_\alpha(t-\tau)d\tau$, $I'_x(t) = \int_0^t A_4(\tau)F_g(t-\tau)d\tau$, $I_y(t) = \int_0^t B_3(\tau)F_\alpha(t-\tau)d\tau$, $I'_y(t) = \int_0^t B_4(\tau)F_g(t-\tau)d\tau$, $I_{\pi_x}(t) = \int_0^t C_3(\tau)F_\alpha(t-\tau)d\tau$, $I'_{\pi_x}(t) = \int_0^t C_4(\tau)F_g(t-\tau)d\tau$, $I_{\pi_y}(t) = \int_0^t D_3(\tau)F_\alpha(t-\tau)d\tau$, and $I'_{\pi_y}(t) = \int_0^t D_4(\tau)F_g(t-\tau)d\tau$ and the coefficients $A_i(t)$, $B_i(t)$, $C_i(t)$, and $D_i(t)(i=1,2,3,4)$ are given in Appendix A.

III. TRANSPORT COEFFICIENTS

In order to determine the transport coefficients, we use the solution (15). Averaging them over the whole system and taking the time derivative, we obtain the following system of equations for the first moments

$$\langle \dot{x}(t) \rangle = \frac{\langle \pi_x(t) \rangle}{\mu},$$

$$\langle \dot{y}(t) \rangle = \frac{\langle \pi_y(t) \rangle}{\mu},$$

$$\langle \dot{\pi}_x(t) \rangle = -\lambda_{\pi_x}(t)\langle \pi_x(t) \rangle + \rho_x(t)\langle \pi_y(t) \rangle - c_x(t)\langle x(t) \rangle + \delta_x(t)\langle y(t) \rangle,$$

$$\langle \dot{\pi}_y(t) \rangle = -\lambda_{\pi_y}(t)\langle \pi_y(t) \rangle + \rho_y(t)\langle \pi_x(t) \rangle - c_y(t)\langle y(t) \rangle + \delta_y(t)\langle x(t) \rangle, \quad (16)$$

where the time-dependent coefficients $\lambda_{\pi_x}(t)$, $\lambda_{\pi_y}(t)$, $\rho_x(t)$, $\rho_y(t)$, $c_x(t)$, $c_y(t)$, $\delta_x(t)$, and $\delta_y(t)$ are presented in Appendix B. The coefficients $\lambda_{\pi_{x,y}}$ are related to the friction coefficients. The renormalized stiffnesses are $c_{x,y}$. If we come back from the variables π_x and π_y to the canonically conjugated moments p_x and p_y, then

$$\langle \dot{x}(t) \rangle = \frac{\langle p_x(t) \rangle}{\mu} + \omega_L \langle y(t) \rangle /2,$$

$$\langle \dot{y}(t) \rangle = \frac{\langle p_y(t) \rangle}{\mu} - \omega_L \langle x(t) \rangle /2,$$

$$\langle \dot{p}_x(t) \rangle = -\lambda_{p_x}(t)\langle p_x(t) \rangle + \tilde{\rho}_x(t)\langle p_y(t) \rangle - \tilde{c}_x(t)\langle x(t) \rangle + \tilde{\delta}_x(t)\langle y(t) \rangle,$$

$$\langle \dot{p}_y(t) \rangle = -\lambda_{p_y}(t)\langle p_y(t) \rangle + \tilde{\rho}_y(t)\langle p_x(t) \rangle - \tilde{c}_y(t)\langle y(t) \rangle + \tilde{\delta}_y(t)\langle x(t) \rangle, \quad (17)$$

where $\lambda_{p_x}(t) = \lambda_{\pi_x}(t)$, $\lambda_{p_y}(t) = \lambda_{\pi_y}(t)$, $\tilde{\rho}_x(t) = \rho_x(t) - \omega_L/2$, $\tilde{\rho}_y(t) = \rho_y(t) + \omega_L/2$, $\tilde{\delta}_x(t) = \delta_x(t) - \lambda_{\pi_x}(t)\mu\omega_L/2$, $\tilde{\delta}_y(t) = \delta_y(t) + \lambda_{\pi_y}(t)\mu\omega_L/2$, $\tilde{c}_x(t) = c_x(t) + \rho_x(t)\mu\omega_L/2 - \mu\omega_L^2/4$, and $\tilde{c}_y(t) = c_y(t) - \rho_y(t)\mu\omega_L/2 - \mu\omega_L^2/4$. From the structure of Eqs. (16) and (17) it is seen that the dynamics is governed by the nonstationary coefficients.

The equations for the second moments (variances),

$$\sigma_{q_i q_j}(t) = \frac{1}{2}\langle q_i(t)q_j(t) + q_j(t)q_i(t) \rangle - \langle q_i(t) \rangle \langle q_j(t) \rangle,$$

where $q_i = x$, y, π_x, or π_y $(i=1-4)$, are

$$\dot{\sigma}_{xx}(t) = \frac{2\sigma_{x\pi_x}(t)}{\mu}, \quad \dot{\sigma}_{yy}(t) = \frac{2\sigma_{y\pi_y}(t)}{\mu},$$

$$\dot{\sigma}_{xy}(t) = \frac{\sigma_{x\pi_y}(t)}{\mu} + \frac{\sigma_{y\pi_x}(t)}{\mu},$$

$$\dot{\sigma}_{x\pi_y}(t) = -\lambda_{\pi_y}(t)\sigma_{x\pi_y}(t) + \rho_y(t)\sigma_{x\pi_x}(t) - c_y(t)\sigma_{xy}(t) + \delta_y(t)\sigma_{xx}(t) + \frac{\sigma_{\pi_x\pi_y}(t)}{\mu} + 2D_{x\pi_y}(t),$$

$$\dot{\sigma}_{x\pi_x}(t) = -\lambda_{\pi_x}(t)\sigma_{x\pi_x}(t) + \rho_x(t)\sigma_{x\pi_y}(t) - c_x(t)\sigma_{xx}(t) + \delta_x(t)\sigma_{xy}(t) + \frac{\sigma_{\pi_x\pi_x}(t)}{\mu} + 2D_{x\pi_x}(t),$$

$$\dot{\sigma}_{y\pi_x}(t) = -\lambda_{\pi_x}(t)\sigma_{y\pi_x}(t) + \rho_x(t)\sigma_{y\pi_y}(t) - c_x(t)\sigma_{xy}(t) + \delta_x(t)\sigma_{yy}(t) + \frac{\sigma_{\pi_x\pi_y}(t)}{\mu} + 2D_{y\pi_x}(t),$$

$$\dot{\sigma}_{y\pi_y}(t) = -\lambda_{\pi_y}(t)\sigma_{y\pi_y}(t) + \rho_y(t)\sigma_{y\pi_x}(t) - c_y(t)\sigma_{yy}(t) + \delta_y(t)\sigma_{xy}(t) + \frac{\sigma_{\pi_y\pi_y}(t)}{\mu} + 2D_{y\pi_y}(t),$$

$$\dot{\sigma}_{\pi_x\pi_x}(t) = -2\lambda_{\pi_x}(t)\sigma_{\pi_x\pi_x}(t) + 2\rho_x(t)\sigma_{\pi_x\pi_y}(t) - 2c_x(t)\sigma_{x\pi_x}(t) + 2\delta_x(t)\sigma_{y\pi_x}(t) + 2D_{\pi_x\pi_x}(t),$$

$$\dot{\sigma}_{\pi_y\pi_y}(t) = -2\lambda_{\pi_y}(t)\sigma_{\pi_y\pi_y}(t) + 2\rho_y(t)\sigma_{\pi_x\pi_y}(t) - 2c_y(t)\sigma_{y\pi_y}(t) + 2\delta_y(t)\sigma_{x\pi_y}(t) + 2D_{\pi_y\pi_y}(t),$$

$$\dot{\sigma}_{\pi_x\pi_y}(t) = -(\lambda_{\pi_x}(t) + \lambda_{\pi_y}(t))\sigma_{\pi_x\pi_y}(t) + \rho_x(t)\sigma_{\pi_y\pi_y}(t) + \rho_y(t)\sigma_{\pi_x\pi_x}(t) - c_x(t)\sigma_{x\pi_y}(t) - c_y(t)\sigma_{y\pi_x}(t) + \delta_x(t)\sigma_{y\pi_y}(t) + \delta_y(t)\sigma_{x\pi_x}(t) + 2D_{\pi_x\pi_y}(t). \quad (18)$$

So we have obtained the Markovian-type (local in time) equations for the first and second moments, but with the transport coefficients depending explicitly on time. The time-dependent diffusion coefficients $D_{q_iq_j}(t)$ are determined as

$$D_{\pi_x\pi_x}(t) = \lambda_{\pi_x}(t)J_{\pi_x\pi_x}(t) - \rho_x(t)J_{\pi_x\pi_y}(t) + c_x(t)J_{x\pi_x}(t) - \delta_x(t)J_{y\pi_x}(t) + \frac{1}{2}\dot{J}_{\pi_x\pi_x}(t),$$

$$D_{\pi_y\pi_y}(t) = \lambda_{\pi_y}(t)J_{\pi_y\pi_y}(t) - \rho_y(t)J_{\pi_x\pi_y}(t) + c_y(t)J_{y\pi_y}(t) - \delta_y(t)J_{x\pi_y}(t) + \frac{1}{2}\dot{J}_{\pi_y\pi_y}(t),$$

$$D_{\pi_x\pi_y}(t) = -\frac{1}{2}\{-[\lambda_{\pi_x}(t) + \lambda_{\pi_y}(t)]J_{\pi_x\pi_y}(t) + \rho_x(t)J_{\pi_y\pi_y}(t) + \rho_y(t)J_{\pi_x\pi_x}(t) - c_x(t)J_{x\pi_y}(t) - c_y(t)J_{y\pi_x}(t) + \delta_x(t)J_{y\pi_y}(t) + \delta_y(t)J_{x\pi_x}(t) - \dot{J}_{\pi_x\pi_y}(t)\},$$

$$D_{x\pi_y}(t) = -\frac{1}{2}\left(-\lambda_{\pi_y}(t)J_{x\pi_y}(t) + \rho_y(t)J_{x\pi_x}(t) - c_y(t)J_{xy}(t) + \delta_y(t)J_{xx}(t) + \frac{J_{\pi_x\pi_y}(t)}{\mu} - \dot{J}_{x\pi_y}(t)\right),$$

$$D_{y\pi_x}(t) = -\frac{1}{2}\left(-\lambda_{\pi_x}(t)J_{y\pi_x}(t) + \rho_x(t)J_{y\pi_y}(t) - c_x(t)J_{xy}(t) + \delta_x(t)J_{yy}(t) + \frac{J_{\pi_x\pi_y}(t)}{\mu} - \dot{J}_{y\pi_x}(t)\right),$$

$$D_{x\pi_x}(t) = -\frac{1}{2}\left(-\lambda_{\pi_x}(t)J_{x\pi_x}(t) + \rho_x(t)J_{x\pi_y}(t) - c_x(t)J_{xx}(t) + \delta_x(t)J_{xy}(t) + \frac{J_{\pi_x\pi_x}(t)}{\mu} - \dot{J}_{x\pi_x}(t)\right),$$

$$D_{y\pi_y}(t) = -\frac{1}{2}\left(-\lambda_{\pi_y}(t)J_{y\pi_y}(t) + \rho_y(t)J_{y\pi_x}(t) - c_y(t)J_{yy}(t) + \delta_y(t)J_{xy}(t) + \frac{J_{\pi_y\pi_y}(t)}{\mu} - \dot{J}_{y\pi_y}(t)\right). \quad (19)$$

Here, $\dot{J}_{q_iq_j}(t) = dJ_{q_iq_j}(t)/dt$. In our treatment $D_{xx}=0$, $D_{yy}=0$, and $D_{xy}=0$ because there are no random forces for the x and y coordinates in Eqs. (9). If $\omega_L=0$, then $D_{y\pi_x}(t)=D_{x\pi_y}(t)=D_{\pi_x\pi_y}(t)=0$. In Eqs. (19) we use the following notation:

$$J_{xx}(t) = \langle\langle I_x(t)I_x(t) + I'_x(t)I'_x(t)\rangle\rangle,$$

$$J_{yy}(t) = \langle\langle I_y(t)I_y(t) + I'_y(t)I'_y(t)\rangle\rangle,$$

$$J_{\pi_x\pi_x}(t) = \langle\langle I_{\pi_x}(t)I_{\pi_x}(t) + I'_{\pi_x}(t)I'_{\pi_x}(t)\rangle\rangle,$$

$$J_{\pi_y\pi_y}(t) = \langle\langle I_{\pi_y}(t)I_{\pi_y}(t) + I'_{\pi_y}(t)I'_{\pi_y}(t)\rangle\rangle,$$

$$J_{\pi_x\pi_y}(t) = \langle\langle I_{\pi_x(t)}I_{\pi_y(t)} + I'_{\pi_x(t)}I'_{\pi_y(t)}\rangle\rangle,$$

$$J_{xy}(t) = \langle\langle I_x(t)I_y(t) + I'_x(t)I'_y(t)\rangle\rangle,$$

$$J_{x\pi_x}(t) = \langle\langle I_x(t)I_{\pi_x}(t) + I'_x(t)I'_{\pi_x}(t)\rangle\rangle,$$

$$J_{y\pi_y}(t) = \langle\langle I_y(t)I_{\pi_y}(t) + I'_y(t)I'_{\pi_y}(t)\rangle\rangle,$$

$$J_{x\pi_y}(t) = \langle\langle I_x(t)I_{\pi_y}(t) + I'_x(t)I'_{\pi_y}(t)\rangle\rangle, \quad (20)$$

$$J_{y\pi_x}(t) = \langle\langle I_y(t)I_{\pi_x}(t) + I'_y(t)I'_{\pi_x}(t)\rangle\rangle.$$

The explicit expressions for $J_{q_iq_j}(t)$ are presented in Appendix C.

At $t\to\infty$ the system reaches the equilibrium state ($\dot{\sigma}_{q_iq_j}=0$). Taking zeros in the left parts of Eqs. (18) at $t\to\infty$, we get a linear system of equations which establishes the one-to-one correspondence between the asymptotic variances and asymptotic diffusion coefficients:

$$D_{\pi_x\pi_x}(\infty) = \lambda_{\pi_x}(\infty)\sigma_{\pi_x\pi_x}(\infty) - \delta_x(\infty)\sigma_{y\pi_x}(\infty),$$

$$D_{\pi_y\pi_y}(\infty) = \lambda_{\pi_y}(\infty)\sigma_{\pi_y\pi_y}(\infty) - \delta_y(\infty)\sigma_{x\pi_y}(\infty),$$

$$D_{\pi_x\pi_y}(\infty) = \frac{1}{2}[c_x(\infty)\sigma_{x\pi_y}(\infty) + c_y(\infty)\sigma_{y\pi_x}(\infty) - \rho_x(\infty)\sigma_{\pi_y\pi_y}(\infty) - \rho_y(\infty)\sigma_{\pi_x\pi_x}(\infty)],$$

$$D_{x\pi_y}(\infty) = \frac{1}{2}[\lambda_{\pi_y}(\infty)\sigma_{x\pi_y}(\infty) - \delta_y(\infty)\sigma_{xx}(\infty)],$$

$$D_{y\pi_x}(\infty) = \frac{1}{2}[\lambda_{\pi_x}(\infty)\sigma_{y\pi_x}(\infty) - \delta_x(\infty)\sigma_{yy}(\infty)],$$

$$D_{x\pi_x}(\infty) = \frac{1}{2}\left(c_x(\infty)\sigma_{xx}(\infty) - \rho_x(\infty)\sigma_{x\pi_y}(\infty) - \frac{1}{\mu}\sigma_{\pi_x\pi_x}(\infty)\right),$$

$$D_{y\pi_y}(\infty)=\frac{1}{2}\left(c_y(\infty)\sigma_{yy}(\infty)-\rho_y(\infty)\sigma_{y\pi_x}(\infty)-\frac{1}{\mu}\sigma_{\pi_y\pi_y}(\infty)\right). \quad (21)$$

Comparing Eqs. (18) and (19), we obtain that $\sigma_{q_iq_j}(\infty)=J_{q_iq_j}(\infty)$. The explicit expressions for $\sigma_{q_iq_j}(\infty)$ are given in Appendix C. In the axisymmetric case ($\omega_x=\omega_y$ or $c_x=c_y$) with $\lambda_x^0=\lambda_y^0$, $\delta_x(\infty)=-\delta_y(\infty)$, $\rho_x(\infty)=-\rho_y(\infty)$, $D_{\pi_x\pi_x}(\infty)=D_{\pi_y\pi_y}(\infty)$, $D_{x\pi_y}(\infty)=-D_{y\pi_x}(\infty)$, $D_{\pi_x\pi_y}(\infty)=0$, $\sigma_{\pi_x\pi_x}(\infty)=\sigma_{\pi_y\pi_y}(\infty)$, $\sigma_{xx}(\infty)=\sigma_{yy}(\infty)$, and $\sigma_{x\pi_y}(\infty)=-\sigma_{y\pi_x}(\infty)$.

Using Eqs. (19) and the transformations between the variances in different coordinate systems, we obtain the asymptotic diffusion coefficients for the coordinates x and y and canonically conjugated moments p_x and p_y:

$$D_{p_xp_x}(\infty)=\lambda_{p_x}(\infty)\sigma_{p_xp_x}(\infty)-\tilde{\delta}_x(\infty)\sigma_{yp_x}(\infty),$$

$$D_{p_yp_y}(\infty)=\lambda_{p_y}(\infty)\sigma_{p_yp_y}(\infty)-\tilde{\delta}_y(\infty)\sigma_{xp_y}(\infty),$$

$$D_{p_xp_y}(\infty)=\frac{1}{2}[\tilde{c}_x(\infty)\sigma_{xp_y}(\infty)+\tilde{c}_y(\infty)\sigma_{yp_x}(\infty)-\tilde{\rho}_x(\infty)\sigma_{p_yp_y}(\infty)-\tilde{\rho}_y(\infty)\sigma_{p_xp_x}(\infty)],$$

$$D_{xp_y}(\infty)=\frac{1}{2}[\lambda_{p_y}(\infty)\sigma_{xp_y}(\infty)-\tilde{\delta}_y(\infty)\sigma_{xx}(\infty)],$$

$$D_{yp_x}(\infty)=\frac{1}{2}[\lambda_{p_x}(\infty)\sigma_{yp_x}(\infty)-\tilde{\delta}_x(\infty)\sigma_{yy}(\infty)],$$

$$D_{xp_x}(\infty)=\frac{1}{2}\left(\tilde{c}_x(\infty)\sigma_{xx}(\infty)-\frac{1}{\mu}\sigma_{p_xp_x}(\infty)-\tilde{\rho}_x(\infty)\sigma_{xp_y}(\infty)-\frac{1}{2}\omega_L\sigma_{yp_x}(\infty)\right),$$

$$D_{yp_y}(\infty)=\frac{1}{2}\left(\tilde{c}_y(\infty)\sigma_{yy}(\infty)-\frac{1}{\mu}\sigma_{p_yp_y}(\infty)-\tilde{\rho}_y(\infty)\sigma_{yp_x}(\infty)+\frac{1}{2}\omega_L\sigma_{xp_y}(\infty)\right). \quad (22)$$

In the axisymmetric case with $\lambda_x^0=\lambda_y^0$, $\tilde{\delta}_x(\infty)=-\tilde{\delta}_y(\infty)$, $\tilde{\rho}_x(\infty)=-\tilde{\rho}_y(\infty)$, $D_{p_xp_x}(\infty)=D_{p_yp_y}(\infty)$, $D_{xp_x}(\infty)=D_{yp_y}(\infty)$, and $D_{xp_y}(\infty)=-D_{yp_x}(\infty)$.

If $\tilde{\delta}_x(\infty)=\lambda_{p_x}(\infty)\mu\omega_L/2$, $\tilde{\delta}_y(\infty)=-\lambda_{p_y}(\infty)\mu\omega_L/2$ ($\delta_x(\infty)=\delta_y(\infty)=0$) in Eqs. (22), then the asymptotic diffusion and friction coefficients are connected by the following fluctuation-dissipation relations

$$D_{p_xp_x}(\infty)=\lambda_{p_x}(\infty)[\sigma_{p_xp_x}(\infty)+\mu\omega_L\sigma_{yp_x}(\infty)/2],$$

$$D_{p_yp_y}(\infty)=\lambda_{p_y}(\infty)[\sigma_{p_yp_y}(\infty)-\mu\omega_L\sigma_{xp_y}(\infty)/2],$$

$$D_{yp_x}(\infty)=\frac{1}{2}\lambda_{p_x}(\infty)[\sigma_{yp_x}(\infty)+\mu\omega_L\sigma_{yy}(\infty)/2],$$

$$D_{xp_y}(\infty)=\frac{1}{2}\lambda_{p_y}(\infty)[\sigma_{xp_y}(\infty)-\mu\omega_L\sigma_{xx}(\infty)/2].$$

IV. ILLUSTRATIVE CALCULATIONS AND DISCUSSION

As shown, the diffusion and friction coefficients depend on the parameters ω_x, ω_y, λ_x^0, λ_y^0, and γ. The values of ω_x, ω_y, λ_x^0, and λ_y^0 are fixed so that in the absence of magnetic field ($\omega_L=0$) they correspond to the certain asymptotic values $\hbar\tilde{\omega}_0=1$ MeV, $\lambda_{p_x}(\infty)$, and $\lambda_{p_y}(\infty)$. In the asymptotic, an isotropic two-dimensional oscillator is assumed. In our illustrative calculations the dynamical collective variables are the quadrupole and octupole deformation parameters which describe the surface vibrations in a heavy atomic nucleus [19]. The motion in quadrupole and octupole deformation parameters produces small oscillations around the equilibrium nuclear shape. The excited vibrational nuclear states are populated in the deep-inelastic collisions between heavy ions. One can measure the widths (a few MeV), which are related to the friction coefficient λ_p, of these states and the transitions from these states to other nuclear states. We choose the characteristic frequency $\hbar\tilde{\omega}_0=1$ MeV and mass parameter $\mu=448m_0$ (m_0 is the nucleon mass) for this type of collective quantum motion. The oscillations of the symmetric dinuclear system in mass asymmetry coordinates and in deformation parameters of the fragments have similar characteristics [20]. We fix the asymptotic frequency of the collective oscillator and express all energetic characteristics in units of this frequency. The value of γ should be taken to fulfill the condition $\gamma\gg\tilde{\omega}_0$. We set $\hbar\gamma=12$ MeV.

In order to come back from the variables π_x and π_y to the canonically conjugated moments p_x and p_y, the following transformations are used for the variances:

$$\sigma_{p_xp_x}(t)=\sigma_{\pi_x\pi_x}(t)-\mu\omega_L\sigma_{y\pi_x}(t)+(\mu\omega_L/2)^2\sigma_{yy}(t),$$

$$\sigma_{p_yp_y}(t)=\sigma_{\pi_y\pi_y}(t)+\mu\omega_L\sigma_{x\pi_y}(t)+(\mu\omega_L/2)^2\sigma_{xx}(t),$$

$$\sigma_{p_xp_y}(t)=\sigma_{\pi_x\pi_y}(t)+(\mu\omega_L/2)[\sigma_{x\pi_x}(t)-\sigma_{y\pi_y}(t)]-(\mu\omega_L/2)^2\sigma_{xy}(t),$$

$$\sigma_{xp_x}(t)=\sigma_{x\pi_x}(t)-(\mu\omega_L/2)\sigma_{xy}(t),$$

$$\sigma_{yp_y}(t)=\sigma_{y\pi_y}(t)+(\mu\omega_L/2)\sigma_{xy}(t),$$

$$\sigma_{xp_y}(t)=\sigma_{x\pi_y}(t)+(\mu\omega_L/2)\sigma_{xx}(t),$$

$$\sigma_{yp_x}(t)=\sigma_{y\pi_x}(t)-(\mu\omega_L/2)\sigma_{yy}(t), \quad (23)$$

and for the friction and diffusion coefficients,

$$\lambda_{p_x}(t)=\lambda_{\pi_x}(t),\ \lambda_{p_y}(t)=\lambda_{\pi_y}(t),$$

$$D_{p_xp_x}(t)=D_{\pi_x\pi_x}(t)-\mu\omega_LD_{y\pi_x}(t),$$

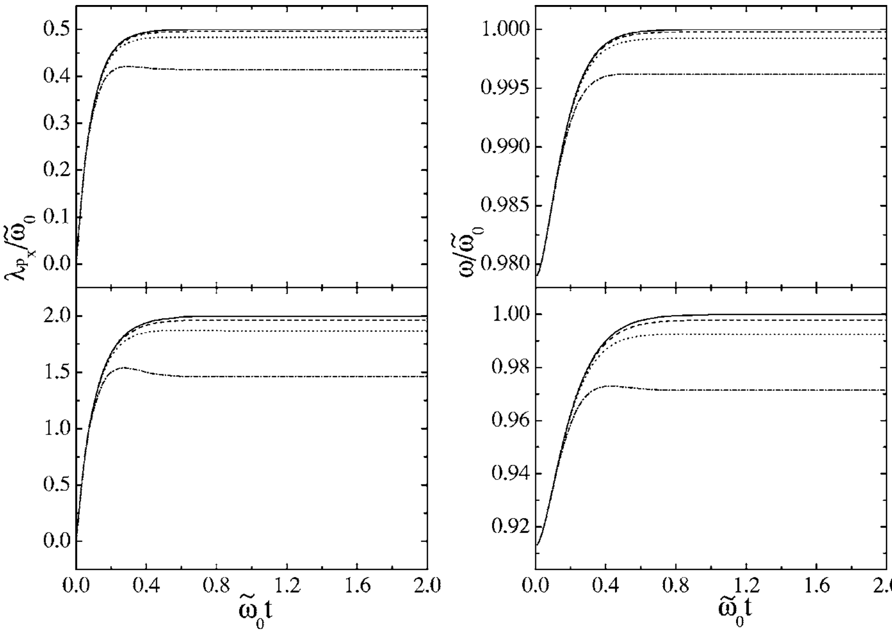

FIG. 1. Calculated time dependence of the friction coefficient λ_{p_x} and of the collective frequency ω at $\lambda_x^0=\lambda_y^0=0.09$ and $\omega_x/\tilde{\omega}_0=\omega_y/\tilde{\omega}_0=2.6$ (upper parts) and at $\lambda_x^0=\lambda_y^0=0.2$ and $\omega_x/\tilde{\omega}_0=\omega_y/\tilde{\omega}_0=4.6$ (lower parts). The results for $\omega_L/\tilde{\omega}_0=0$, 1.0, 2.0, and 5.0 are presented by solid, dashed, dotted, and dash-dotted lines, respectively. The upper and lower parts correspond to $\lambda_{p_x}(\infty)/\tilde{\omega}_0=0.5$ and 2, respectively, at $\omega_L=0$.

$$D_{p_yp_y}(t)=D_{\pi_y\pi_y}(t)+\mu\omega_L D_{x\pi_y}(t),$$

$$D_{p_xp_y}(t)=D_{\pi_x\pi_y}(t)+(\mu\omega_L/2)[D_{x\pi_x}(t)-D_{y\pi_y}(t)],$$

$$D_{xp_x}(t)=D_{x\pi_x}(t), D_{yp_y}(t)=D_{y\pi_y}(t),$$

$$D_{xp_y}(t)=D_{x\pi_y}(t), D_{yp_x}(t)=D_{y\pi_x}(t). \tag{24}$$

In our calculations, the initial Gaussian distribution has $\sqrt{\sigma_{xx}(0)}=\sqrt{\sigma_{yy}(0)}=0.2(\mu\tilde{\omega}_0/\hbar)^{1/2}$. Using the uncertainty relation with $\sigma_{xp_x}(0)=0$, $\sigma_{yp_y}(0)=0$, $\sigma_{xp_y}(0)=0$, $\sigma_{yp_x}(0)=0$, $\sigma_{xy}(0)=0$, and $\sigma_{p_xp_y}(0)=0$, the variances $\sigma_{p_xp_x}(0)$ and $\sigma_{p_yp_y}(0)$ are chosen as follows: $\sigma_{p_xp_x}(0)=\hbar^2/[4\sigma_{xx}(0)]$ and $\sigma_{p_yp_y}(0)=\hbar^2/[4\sigma_{yy}(0)]$.

A. Time-dependent friction and diffusion coefficients

The time evolution of the friction coefficient and of the collective frequency at different strengths of the magnetic field is demonstrated in Fig. 1. Since λ_{p_x}, λ_{p_y} and ω_x, ω_y have the same time behavior because of the axisymmetry, we show only λ_{p_x} and $\omega_x=\omega$. From the results in Fig. 1 one can conclude that if the magnetic field acts on a quantum particle only through the Lorentz force term, then the asymptotic value of the friction coefficient decreases with increasing value of ω_L. Usually, for fermion systems the resistance increases with increasing strength of the magnetic field. However, for the bosonic system considered it may decrease. There are experimental findings of a decreasing resistance in the magnetic field [5]. Although this effect is explained within a quantum mechanical treatment, it results from our transport theory as well. It should be noted that our result is in agreement with the theorem of Ref. [21]: if the influence of the external magnetic field on the electrons is only through its contribution to the Lorentz force, then the electrical conductivity of a metal is a monotonically nonincreasing function of the magnitude of magnetic field.

The time evolutions of the diffusion coefficients $D_{p_xp_x}$, D_{xp_x}, D_{xp_y}, and $D_{p_xp_y}$ are shown in Figs. 2–5. These coefficients are initially equal to zero, and in some transient time they reach their asymptotic values. As one can see, the transient time increases with ω_L. While the asymptotic value of $D_{p_xp_x}$ increases with ω_L in the case of weak coupling, in the case of strong coupling and high temperatures it may decrease with respect to the value of $D_{p_xp_x}(\infty)$ at $\omega_L=0$ (see lower part of right side of Fig. 2). This correlates with the time behavior of friction coefficient λ_{p_x} and variances $\sigma_{p_xp_x}$ and $\sigma_{xp_y}=-\sigma_{yp_x}$. The absolute value of $D_{xp_x}(\infty)$ decreases with increasing ω_L and approaches nearly zero in Fig. 3. $D_{xp_y}=0$ in the absence of a magnetic field, but in the field it becomes nonzero with a negative asymptotic value (Fig. 4). The asymptotic value of $|D_{xp_y}|$ increases with ω_L and decreases with increasing temperature. The value of $D_{p_xp_y}$ is equal to zero at $\lambda_x^0=\lambda_y^0$ and becomes negative (positive) at $\lambda_x^0>\lambda_y^0$ ($\lambda_x^0<\lambda_y^0$) because

$$D_{p_xp_y}\approx\frac{\mu^2(\tilde{\omega}_0^2+\omega_L^2/2)\omega_L}{4}[\sigma_{xx}(\infty)-\sigma_{yy}(\infty)]$$

[see Eqs. (22)] and $\sigma_{xx}(\infty)<\sigma_{yy}(\infty)$ at $\lambda_x^0>\lambda_y^0$ [$\sigma_{xx}(\infty)>\sigma_{yy}(\infty)$ at $\lambda_x^0<\lambda_y^0$]. This is also clear from the expression for $D_{p_xp_y}$ [see Eqs. (24)] where $D_{\pi_x\pi_y}$ is quite small and, thus, $D_{p_xp_y}$ changes sign after the replacement $x\leftrightarrow y$. The role of nondiagonal components of the diffusion tensor is suppressed with increasing temperature. This statement is also confirmed in Fig. 6 where the dependences of the asymptotic values of the diffusion coefficients on the value of ω_L are presented. The absolute values of D_{xp_x} (D_{xp_y}) become smaller (larger) with increasing strength of magnetic field. In the axisymmetric case with $\lambda_x^0=\lambda_y^0$, $\sigma_{p_xp_x}(\infty)=\sigma_{p_yp_y}(\infty)$, $\sigma_{xx}(\infty)=\sigma_{yy}(\infty)$, and $\sigma_{xp_y}(\infty)=-\sigma_{yp_x}(\infty)$ [in the general case

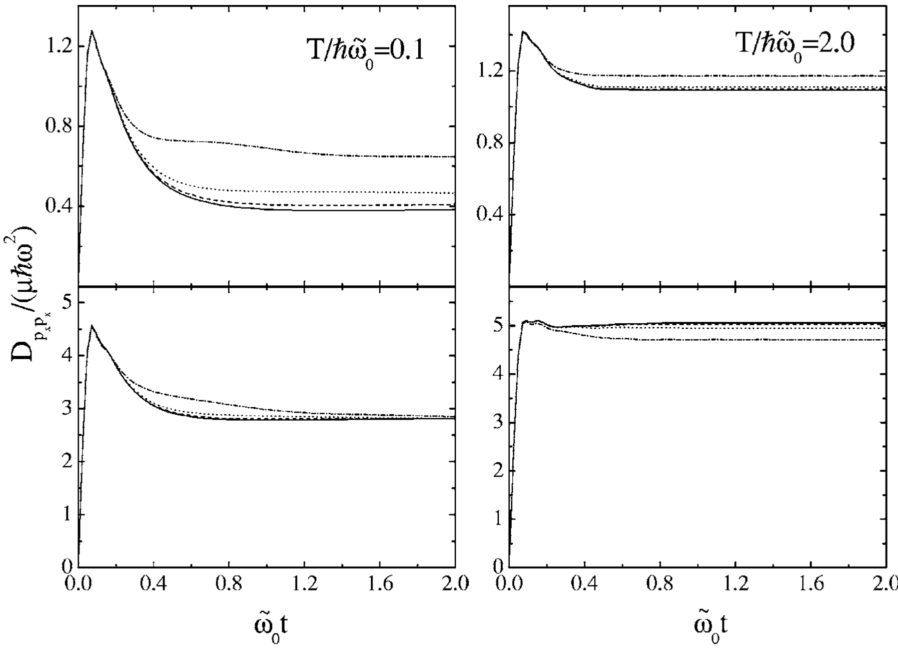

FIG. 2. Calculated time dependence of the diffusion coefficient $D_{p_xp_x}(t)$ at $T/(\hbar\tilde{\omega}_0)=0.1$ (left side) and $T/(\hbar\tilde{\omega}_0)=2$ (right side). The upper and lower parts as well as the notations of the curves correspond to the parameters used in Fig. 1.

($\tilde{c}_x \neq \tilde{c}_y$, $\lambda_{p_x} \neq \lambda_{p_x}$) $\sigma_{xp_y}(\infty) - \mu\omega_L\sigma_{xx}(\infty)/2 = -\sigma_{yp_x}(\infty) - \mu\omega_L\sigma_{yy}(\infty)/2$] and the asymptotes of other variances are equal to zero. The nonzero asymptotic variances, which are presented in Fig. 7, are related to the asymptotics of the diffusion coefficients [see Eqs. (21) and (22)]. For example, while the asymptotic variance in coordinates decreases, the asymptotic variance in momentum and $|\sigma_{yp_x}(\infty)| = \sigma_{xp_y}(\infty)$ increases with ω_L ($\tilde{\delta}_x \approx -\lambda_{p_x}\mu\omega_L/2 < 0$). This explains the rather weak dependence of $D_{p_xp_x}(\infty)$ on ω_L. Since at high temperatures and strong magnetic field $|\sigma_{yp_x}|$ has quite a large asymptotic value (Fig. 7), the value of $D_{p_xp_x}(\infty)$ can slightly decrease with increasing ω_L. At low temperature the value of $\sigma_{xx}(\infty)$ steeply decreases with increasing magnetic field, which leads to a squeeze of the wave packet moving in the heat bath. This effect vanishes at high temperature where the dependence of $\sigma_{xx}(\infty)$ on ω_L is rather weak.

It is well known that dissipation always leads to an enhanced localization of the charged particle when the external magnetic field is zero. Our calculations at low temperature (Fig. 7) show that when the magnetic field is stronger than a certain critical value [$\omega_L \gtrsim (2.5-3)\tilde{\omega}_0$], dissipation [$\lambda_{p_x}(\infty)/\tilde{\omega}_0 = \lambda_{p_y}(\infty)/\tilde{\omega}_0 = 2$ at $\omega_L = 0$] actually delocalizes the charged particle. At high temperatures this unexpected result is not observed. So one can note that the localization is determined by the interplay between the dissipation, external field, and temperature. For the case of an Ohmic heat bath at zero temperature and weak coupling, this interesting phenomenon was also mentioned in Ref. [10].

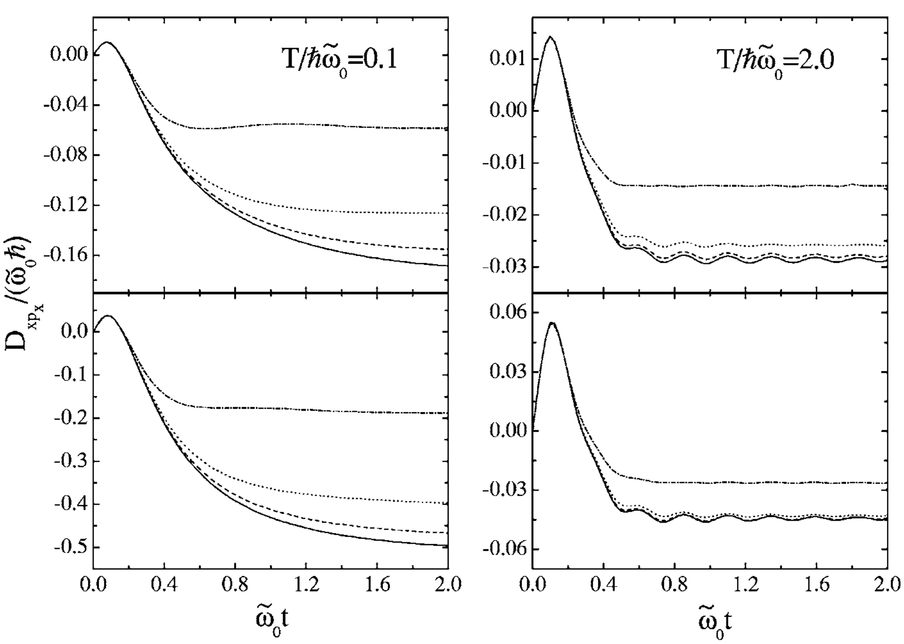

FIG. 3. The same as in Fig. 2, but for the diffusion coefficient $D_{xp_x}(t)$.

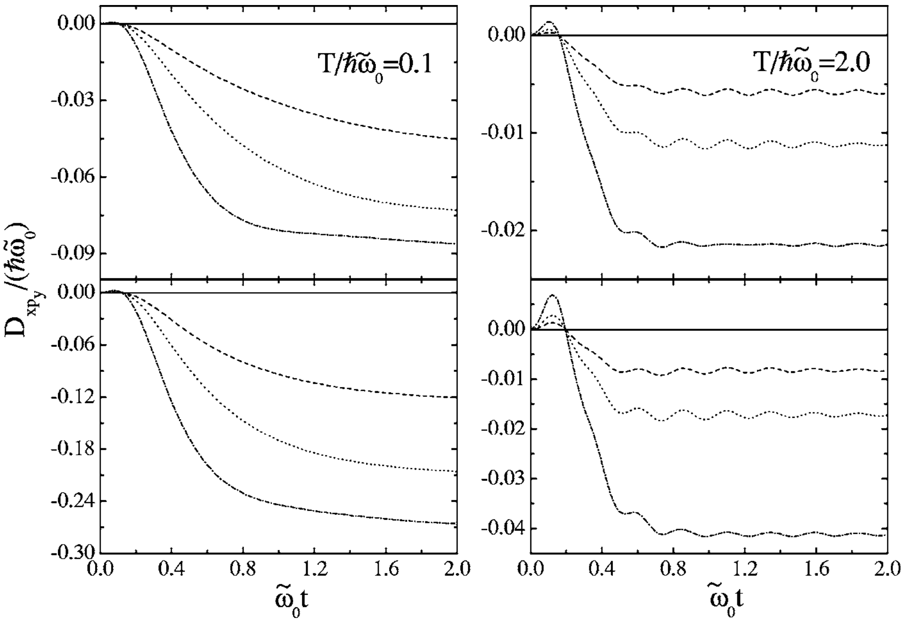

FIG. 4. The same as in Fig. 2, but for the diffusion coefficient $D_{xp_y}(t)$.

B. Comparison of equilibrium variances

In Ref. [7] an equilibrium Wigner function for an isolated two-dimensional isotropic oscillator in a perpendicular constant external magnetic field has been derived. Using this function, the following equilibrium variances have been obtained:

$$\sigma_{\pi_x\pi_x}(\infty) = \frac{\hbar\mu\Omega}{2}\frac{[1+\omega_L^2/(2\Omega)^2]\sinh(\hbar\Omega/T)-[\omega_L/\Omega]\sinh[\hbar\omega_L/(2T)]}{\cosh(\hbar\Omega/T)-\cosh[\hbar\omega_L/(2T)]},$$

$$\sigma_{xx}(\infty) = \frac{\hbar}{2\mu\Omega}\frac{\sinh(\hbar\Omega/T)}{\cosh(\hbar\Omega/T)-\cosh[\hbar\omega_L/(2T)]},$$

$$\sigma_{x\pi_y}(\infty) = -\frac{\hbar}{2}\frac{[\omega_L/(2\Omega)]\sinh(\hbar\Omega/T)-\sinh[\hbar\omega_L/(2T)]}{\cosh(\hbar\Omega/T)-\cosh[\hbar\omega_L/(2T)]},$$

$$\sigma_{\pi_y\pi_y}(\infty) = \sigma_{\pi_x\pi_x}(\infty), \quad \sigma_{yy}(\infty) = \sigma_{xx}(\infty),$$

$$\sigma_{y\pi_x}(\infty) = -\sigma_{x\pi_y}(\infty),$$

$$\sigma_{x\pi_x}(\infty) = 0, \quad \sigma_{xy}(\infty) = 0, \quad \sigma_{y\pi_y}(\infty) = 0, \quad \sigma_{\pi_x\pi_y}(\infty) = 0, \tag{25}$$

where $\Omega=\sqrt{\tilde{\omega}_0{}^2+(\omega_L/2)^2}$ and the friction coefficients are equal to zero. The dependences of these variances and our asymptotic variances on ω_L are compared in Fig. 7. The deviation of our asymptotic variances from the results of Ref. [7] increases with friction. However, this deviation becomes negligible with increasing temperature. Using the variances (25) and some phenomenological assumptions to introduce the dependence on friction, several variants of asymptotic diffusion coefficients were found in Ref. [7]. The reasonable agreement between our asymptotic variances and those in Ref. [7] supports the validity of our calculation of the diffusion coefficients. For the isotropic two-dimensional oscillator and $\lambda_x^0=\lambda_y^0$, in the limit of large time we obtain $\lambda_{p_x}=\lambda_{p_y}=\lambda$, $\rho_x=-\rho_y\approx\omega_L$, $c_x=c_y\approx\mu\tilde{\omega}_0^2$, and $\delta_x=\delta_y\approx 8\times 10^{-3}\mu\lambda\omega_L$, which leads to the same system of equations for the first moments like in Ref. [7] where the time dependence of the transport coefficients is disregarded.

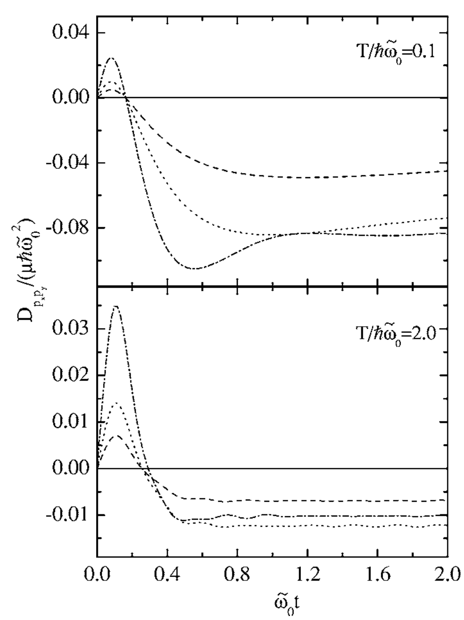

FIG. 5. Calculated time dependence of diffusion coefficient $D_{p_xp_y}(t)$ for λ_x^0=0.09 [$\lambda_{p_x}(\infty)/\tilde{\omega}_0$=0.5 at ω_L=0], λ_y^0=0.13 [$\lambda_{p_y}(\infty)/\tilde{\omega}_0$=1 at ω_L=0], and $\omega_x/\tilde{\omega}_0$=2.6, $\omega_y/\tilde{\omega}_0$=3.5. Upper and lower parts correspond to the indicated temperatures. The results for $\omega_L/\tilde{\omega}_0$=0, 1, 2, and 5 are presented by solid, dashed, dotted, and dash-dotted lines, respectively.

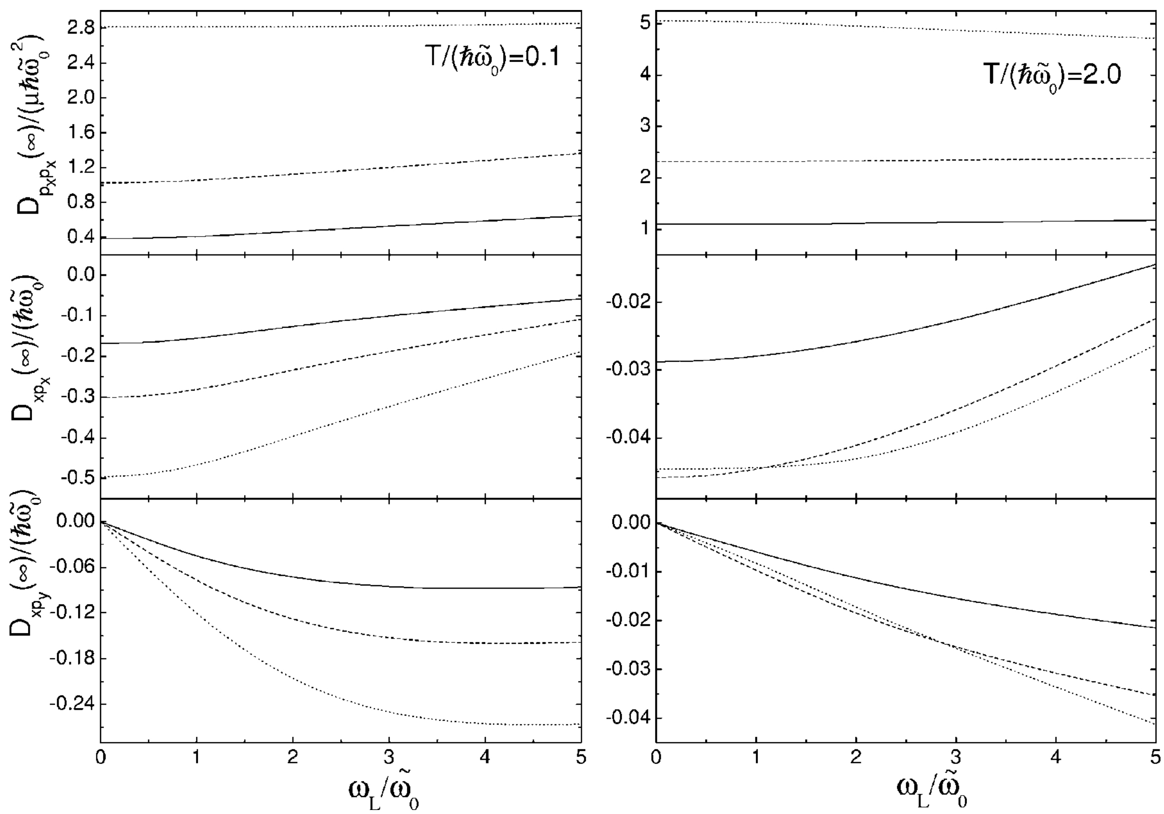

FIG. 6. Dependencies of asymptotics of diffusion coefficients on ω_L. Solid, dashed, and dotted lines correspond to $\lambda_{p_x}(\infty)/\tilde{\omega}_0=\lambda_{p_y}(\infty)/\tilde{\omega}_0=0.5$, 1, and 2, respectively, at $\omega_L=0$.

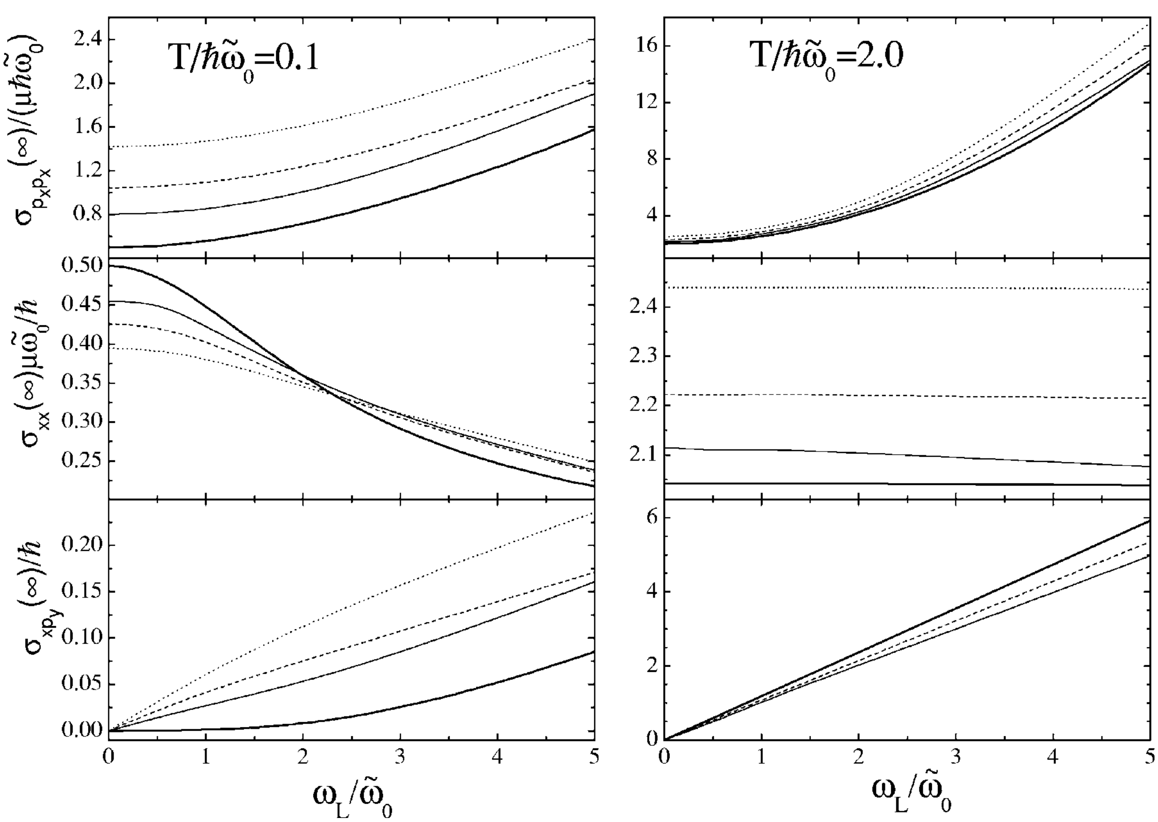

FIG. 7. The same as in Fig. 6, but for the asymptotics of variances. The equilibrium variances obtained in Ref. [7] are presented by thick solid lines.

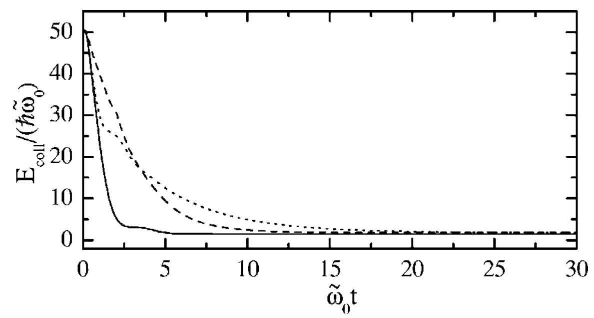

FIG. 8. Time dependence of the total collective energy E_{coll} for $\lambda_{p_x}(\infty)/\tilde{\omega}_0=\lambda_{p_y}(\infty)/\tilde{\omega}_0=0.5$ (at $\omega_L=0$) and $T/(\hbar\tilde{\omega}_0)=0.1$. The initial conditions are given in the text. The results for $\omega_L/\tilde{\omega}_0=0$, 1, 2 are presented by solid, dashed, and dotted lines, respectively.

C. Dissipation of collective energy

The time dependences of the collective energy

$$E_{coll}(t) = E^x_{coll}(t) + E^y_{coll}(t) = \frac{\langle \pi_x^2(t)\rangle}{2\mu} + \frac{c_x(t)\langle x^2(t)\rangle}{2} + \frac{\langle \pi_y^2(t)\rangle}{2\mu} + \frac{c_y(t)\langle y^2(t)\rangle}{2} \quad (26)$$

for different values of magnetic field are shown in Fig. 8. The initial Gaussian distribution is centered at $\langle x(0)\rangle=2$ fm, $\langle y(0)\rangle=2$ fm and $\langle p_x(0)\rangle=10\hbar^2/\text{fm}$, $\langle p_y(0)\rangle=10\hbar^2/\text{fm}$. The magnetic field does not affect the energy of the equilibrium state of the collective subsystem. One can see that the dissipation rate decreases with increasing strength of magnetic field. Therefore, the transient time t_{tr} of $E_{coll}(t)$ [the time in which the value $E_{coll}(t)$ reaches $1.1E_{coll}(\infty)$] increases with ω_L (Fig. 9). Since the oscillations of $E^x_{coll}(t)$ and $E^y_{coll}(t)$ suppress each other in Eq. (26), the time dependence of E_{coll} is rather smooth. Note that there are no qualitative differences between $E_{coll}(t)$ calculated for various frictions and temperatures.

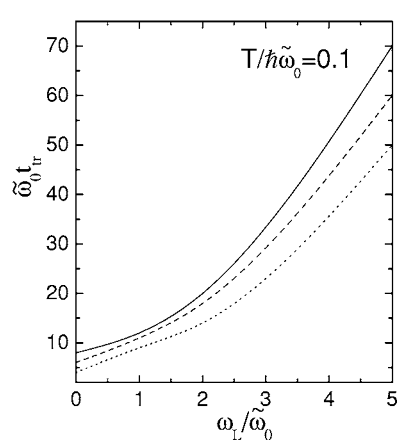

FIG. 9. Dependence of transient time t_{tr} on ω_L. Solid, dashed, and dotted lines correspond to $\lambda_{p_x}(\infty)/\tilde{\omega}_0=\lambda_{p_y}(\infty)/\tilde{\omega}_0=0.5$, 1, and 2, respectively, at $\omega_L=0$.

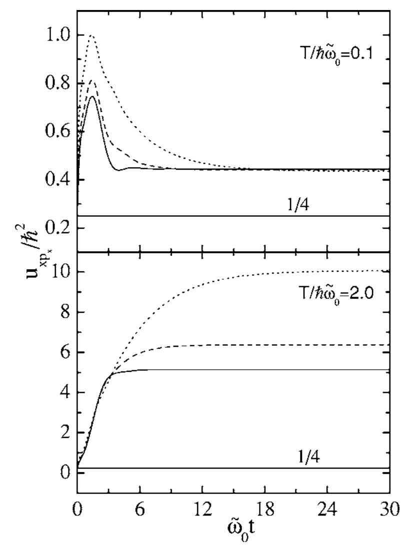

FIG. 10. Time dependence of $u_{xp_x}(t)=\sigma_{xx}(t)\sigma_{p_xp_x}(t)-\sigma^2_{xp_x}(t)$ for $\lambda_{p_x}(\infty)/\tilde{\omega}_0=\lambda_{p_y}(\infty)/\tilde{\omega}_0=2$ (at $\omega_L=0$) and two indicated values of temperature. The results for $\omega_L/\tilde{\omega}_0=0$, 1, and 2 are presented by solid, dashed, and dotted lines, respectively. The horizontal solid lines indicate the values $u_{xp_x}/\hbar^2=1/4$.

D. Uncertainty relation

Since the diffusion coefficients are self-consistently calculated, the uncertainty relation $\sigma(t)=\det\|\sigma_{q_iq_j}(t)\|\geqslant(\hbar^2/4)^2$ or the positivity of the reduced density matrix should be held at any time. In Fig. 10 we show $u_{xp_x}(t)=\sigma_{xx}(t)\sigma_{p_xp_x}(t)-\sigma^2_{xp_x}(t)$ as a function of time. Since in our case $u_{xp_x}(t)=u_{yp_y}(t)$ and, as one can see, the condition $u_{xp_x}(t)>\hbar^2/4$ holds at any time $t>0$, the uncertainty relation is also true at any time. In general, the value of u_{xp_x} increases with temperature. As follows from Fig. 7, the asymptotic of u_{xp_x} is almost independent of ω_L for small temperature and increases with ω_L for large temperature. The rate of $\sigma(t)$ or $u_{xp_x}(t)=u_{yp_y}(t)$ is connected with the rate of linear entropy production, $\dot{S}_{lin}=\hbar^2\dot{\sigma}(t)/\{8[\sigma(t)]^{3/2}\}$ [13]. Therefore, for small T and large ω_L the entropy production is expected to be larger during a short initial time interval. For large temperature and time, the linear entropy S_{lin} increases with the strength of magnetic field.

E. Role of magnetic field in channeling

For the atomic physics problem, we choose the characteristic asymptotic frequency $\hbar\tilde{\omega}_0=0.5$ eV (in the absence of magnetic field) and mass parameter $\mu=4m_0$. These parameters can be related to the channeling of α particles in a monocrystal [22]. The fluctuations of the cross section of the beam in the channel of monocrystal are of interest and are characterized by the value of $\sigma_{rr}(t)=\sigma_{xx}(t)+\sigma_{yy}(t)$. The time dependences $\sigma_{rr}(t)$ are presented in Fig. 11 for various frictions, temperatures, and strengths of magnetic field. One should note the relatively weak dependence on the temperature. For small friction, an increase of ω_L leads to a decrease of the amplitude of oscillations of $\sigma_{rr}(t)$. However, the fre-

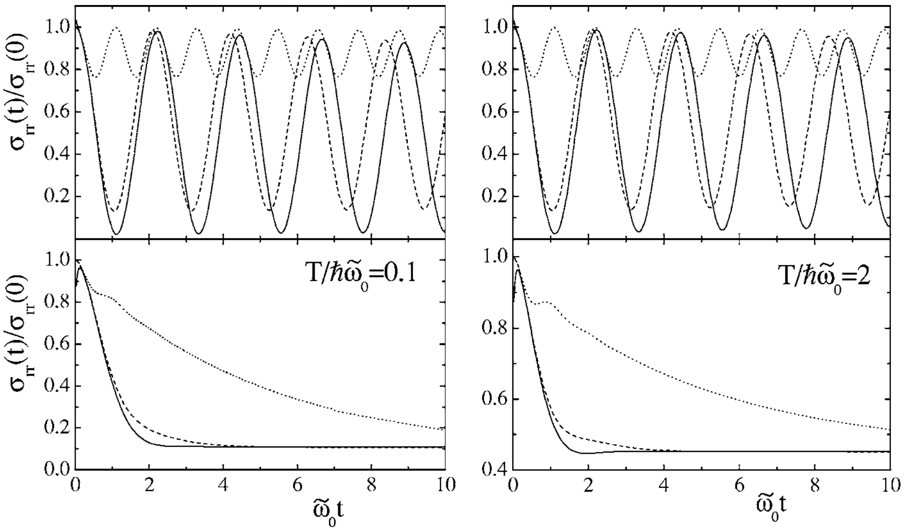

FIG. 11. Time dependence of the value of $\sigma_{rr}(t)/\sigma_{rr}(0)$ at $T/(\hbar\tilde{\omega}_0)=0.1$ (left side) and $T/(\hbar\tilde{\omega}_0)=2$ (right side). The calculations are performed for $\lambda_{p_x}(\infty)/\tilde{\omega}_0=\lambda_{p_y}(\infty)/\tilde{\omega}_0=0.01$ (upper parts) and 2 (lower parts) at $\omega_L=0$. The results for $\omega_L/\tilde{\omega}_0=0$, 1, and 5 are presented by solid, dashed, and dotted lines, respectively.

quency of these oscillation increases. Thus, the magnetic field destroys the effect of superfocusing [23] and stabilizes the transversal dispersion of the beam in the crystal channel. In the case of large friction the magnetic field does not affect much the cross section of the beam besides the initial time interval where the cross section increases with ω_L.

V. SUMMARY

The influence of an external magnetic field on the transport properties of an open quantum system was studied beyond the Markov approximation. Explicit expressions for the time-dependent friction and diffusion coefficients were obtained for a two-dimensional charged quantum harmonic oscillator in a uniform magnetic field. The linear coupling in coordinates to a neutral bosonic heat bath was treated. Our formalism is valid at arbitrary coupling strengths and hence at arbitrary low temperatures. At the initial time interval the magnetic field acts on a quantum particle through its contribution to the Lorentz force. During the process, the dissipation and external magnetic field do affect each other due to the non-Markovian dynamics of the quantum system. One of the central results is that the friction experienced by the transported quantum system is reduced by the presence of an axial magnetic field, leading to reduced energy damping of the system. However, the asymptotic value of this energy is almost independent of ω_L. The results obtained for the asymptotic diffusion coefficients and variances are in qualitative agreement with the results of Refs. [7]. The influence of magnetic field on the dynamics of the system is more pronounced in the case of small temperature at which the magnetic interactions may be used to obtain squeezed wave packet dynamics under certain conditions. With a magnetic field one can regulate the transversal dispersions of the beam in the channel of the crystal. It was shown that the interplay between the dissipation, magnetic field, and temperature leads to interesting phenomena. At low temperature the dissipation reduces the transversal localization of the charged particle when the magnetic field and dissipation are larger than a certain critical values. For the magnetic field and dissipation less than the critical values, the dissipation enhances localization.

For a particle confined by a harmonic potential rotating with frequency ω_{rot}, the Hamiltonian is given by the Eq. (1) with ω_L replaced by ω_{rot}. This Hamiltonian with more complicated collective potential can be used for the description of a Bose-Einstein condensation in a rotating frame [24] and in an environment. Analogous effects as in the uniform magnetic field are expected for a rotating collective system coupled to a bosonic heat bath. For example, the friction coefficient is decreased by rotating frequency ω_{rot} or the dissipation reduces the transversal localization of a quantum particle when ω_{rot} is larger than the certain critical value.

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APPENDIX A

The coefficients in Eqs. (15) are derived as in Ref. [12]:

$$A_1(t)=\sum_{i=1}^{6}\beta_i\{[(\omega_y^2+s_i^2)(s_i+\gamma)-2\lambda_y^0\omega_y\gamma^2][s_i(s_i+\gamma)+2\lambda_x^0\omega_x\gamma]+\omega_L^2 s_i(s_i+\gamma)^2\}e^{s_i t},$$

$$A_2(t)=-\omega_L(\omega_y^2-2\lambda_y^0\omega_y\gamma)\sum_{i=1}^{6}\beta_i(s_i+\gamma)^2e^{s_i t},$$

$$A_3(t)=\frac{1}{\mu}\sum_{i=1}^{6}\beta_i(s_i+\gamma)[(\omega_y^2+s_i^2)(s_i+\gamma)-2\lambda_y^0\omega_y\gamma^2]e^{s_i t},$$

$$A_4(t)=\frac{\omega_L}{\mu}\sum_{i=1}^{6}\beta_i s_i(s_i+\gamma)^2e^{s_i t},$$

$$B_1(t) = -A_2(t)|_{x\leftrightarrow y}, \quad B_2(t) = A_1(t)|_{x\leftrightarrow y},$$

$$B_3(t) = -A_4(t)|_{x\leftrightarrow y}, \quad B_4(t) = A_3(t)|_{x\leftrightarrow y},$$

$$C_1(t) = -\mu^2(\omega_x^2 - 2\lambda_x^0\omega_x\gamma)A_3(t), \quad C_2(t) = -\mu\dot{A}_2(t),$$

$$C_3(t) = \mu\dot{A}_3(t), \quad C_4(t) = \mu\dot{A}_4(t),$$

$$D_1(t) = \mu\dot{B}_1(t), \quad D_2(t) = -\mu^2(\omega_y^2 - 2\lambda_y^0\omega_y\gamma)B_4(t),$$

$$D_3(t) = \mu\dot{B}_3(t), \quad D_4(t) = \mu\dot{B}_4(t). \tag{A1}$$

Here, s_i are the roots of the following equation:

$$[(\omega_x^2 + s_i^2)(s_i + \gamma) - 2\lambda_x^0\omega_x\gamma^2][(\omega_y^2 + s_i^2)(s_i + \gamma) - 2\lambda_y^0\omega_y\gamma^2] + \omega_L^2 s_i^2(s_i + \gamma)^2 = 0 \tag{A2}$$

and $\beta_i = [\Pi_{j\neq i}(s_i - s_j)]^{-1}$ with $i,j = 1-6$. These roots arise when we apply the residue theorem to perform integration in the inverse Laplace transformation.

APPENDIX B

Using Eqs. (15), we write Eqs. (16) for the first moments in which the coefficients after simple algebra are

$$\begin{aligned}\lambda_{\pi_x}(t) = -\{&[B_1(t)\dot{C}_2(t) - B_2(t)\dot{C}_1(t)][A_3(t)D_4(t) - A_4(t)D_3(t)]\\&+ [B_1(t)\dot{C}_3(t) - B_3(t)\dot{C}_1(t)][A_4(t)D_2(t) - A_2(t)D_4(t)]\\&+ [B_1(t)\dot{C}_4(t) - B_4(t)\dot{C}_1(t)][A_2(t)D_3(t) - A_3(t)D_2(t)]\\&+ [B_2(t)\dot{C}_3(t) - B_3(t)\dot{C}_2(t)][A_1(t)D_4(t) - A_4(t)D_1(t)]\\&+ [B_2(t)\dot{C}_4(t) - B_4(t)\dot{C}_2(t)][A_3(t)D_1(t) - A_1(t)D_3(t)]\\&+ [B_3(t)\dot{C}_4(t) - B_4(t)\dot{C}_3(t)][A_1(t)D_2(t)\\&- A_2(t)D_1(t)]\}/I(t),\end{aligned}$$

$$\begin{aligned}\rho_x(t) = \{&[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t)][A_3(t)B_4(t) - A_4(t)B_3(t)]\\&+ [C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t)][A_4(t)B_2(t) - A_2(t)B_4(t)]\\&+ [C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t)][A_2(t)B_3(t) - A_3(t)B_2(t)]\\&+ [C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t)][A_1(t)B_4(t) - A_4(t)B_1(t)]\\&+ [C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t)][A_3(t)B_1(t) - A_1(t)B_3(t)]\\&+ [C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t)][A_1(t)B_2(t)\\&- A_2(t)B_1(t)]\}/I(t),\end{aligned}$$

$$\begin{aligned}c_x(t) = -\{&[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t)][B_3(t)D_4(t) - B_4(t)D_3(t)]\\&+ [C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t)][B_4(t)D_2(t) - B_2(t)D_4(t)]\\&+ [C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t)][B_2(t)D_3(t) - B_3(t)D_2(t)]\\&+ [C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t)][B_1(t)D_4(t) - B_4(t)D_1(t)]\\&+ [C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t)][B_3(t)D_1(t) - B_1(t)D_3(t)]\\&+ [C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t)][B_1(t)D_2(t)\\&- B_2(t)D_1(t)]\}/I(t),\end{aligned}$$

$$\begin{aligned}\delta_x(t) = \{&[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t)][A_4(t)D_3(t) - A_3(t)D_4(t)]\\&+ [C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t)][A_2(t)D_4(t) - A_4(t)D_2(t)]\\&+ [C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t)][A_3(t)D_2(t) - A_2(t)D_3(t)]\\&+ [C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t)][A_4(t)D_1(t) - A_1(t)D_4(t)]\\&+ [C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t)][A_1(t)D_3(t) - A_3(t)D_1(t)]\\&+ [C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t)][A_2(t)D_1(t)\\&- A_1(t)D_2(t)]\}/I(t),\end{aligned}$$

$$\begin{aligned}I(t) = &[B_1(t)D_2(t) - B_2(t)D_1(t)][A_4(t)C_3(t) - A_3(t)C_4(t)]\\&+ [B_1(t)D_3(t) - B_3(t)D_1(t)][A_2(t)C_4(t) - A_4(t)C_2(t)]\\&+ [B_1(t)D_4(t) - B_4(t)D_1(t)][A_3(t)C_2(t) - A_2(t)C_3(t)]\\&+ [B_2(t)D_3(t) - B_3(t)D_2(t)][A_4(t)C_1(t) - A_1(t)C_4(t)]\\&+ [B_2(t)D_4(t) - B_4(t)D_2(t)][A_1(t)C_3(t) - A_3(t)C_1(t)]\\&+ [B_3(t)D_4(t) - B_4(t)D_3(t)][A_2(t)C_1(t) - A_1(t)C_2(t)].\end{aligned}$$

Here, the overdot means the time derivative. The expressions for the coefficients for the y coordinate are obtained from these expressions using the following replacements: $A_i \leftrightarrow B_i$ and $C_i \leftrightarrow D_i$ $(i = 1,2,3,4)$.

APPENDIX C

For the damped quantum two-dimensional fully coupled (FC) oscillator, the expressions for the coefficients $J_{q_iq_j}(t) = J^x_{q_iq_j}(t) + J^y_{q_iq_j}(t)$ follow from Eqs. (20):

$$\begin{aligned}J^x_{xx}(t) = \frac{2\hbar\omega_x\mu\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega + 1]}{\gamma^2 + \omega^2}(a_{ij}\{&[A_3^i(t)A_3^j(t)\\&+ A_3^i(0)A_3^j(0)] - [A_3^i(t)A_3^j(0) + A_3^i(0)A_3^j(t)]\cos(\omega t)\}\\&- b_{ij}[A_3^i(t)A_3^j(0) - A_3^i(0)A_3^j(t)]\sin(\omega t)),\end{aligned}$$

$$\begin{aligned}J^y_{xx}(t) = \frac{2\hbar\omega_y\mu\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega + 1]}{\gamma^2 + \omega^2}(a_{ij}\{&[B_3^i(t)B_3^j(t)\\&+ B_3^i(0)B_3^j(0)] - [B_3^i(t)B_3^j(0) + B_3^i(0)B_3^j(t)]\cos(\omega t)\}\\&- b_{ij}[B_3^i(t)B_3^j(0) - B_3^i(0)B_3^j(t)]\sin(\omega t)),\end{aligned} \tag{C1}$$

$$\begin{aligned}J^x_{yy}(t) = \frac{2\hbar\omega_x\mu\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega + 1]}{\gamma^2 + \omega^2}(a_{ij}\{&[A_4^i(t)A_4^j(t)\\&+ A_4^i(0)A_4^j(0)] - [A_4^i(t)A_4^j(0) + A_4^i(0)A_4^j(t)]\cos(\omega t)\}\\&- b_{ij}[A_4^i(t)A_4^j(0) - A_4^i(0)A_4^j(t)]\sin(\omega t)),\end{aligned}$$

$$J^{y}_{yy}(t)=\frac{2\hbar\omega_y\mu\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[B_4^i(t)B_4^j(t)+B_4^i(0)B_4^j(0)]-[B_4^i(t)B_4^j(0)+B_4^i(0)B_4^j(t)]\cos(\omega t)\}-b_{ij}[B_4^i(t)B_4^j(0)-B_4^i(0)B_4^j(t)]\sin(\omega t)), \tag{C2}$$

$$J^{x}_{xy}(t)=\frac{2\hbar\omega_x\mu\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[A_3^i(t)B_3^j(t)+A_3^i(0)B_3^j(0)]-[A_3^i(t)B_3^j(0)+A_3^i(0)B_3^j(t)]\cos(\omega t)\}-b_{ij}[A_3^i(t)B_3^j(0)-A_3^i(0)B_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{xy}(t)=\frac{2\hbar\omega_y\mu\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[A_4^i(t)B_4^j(t)+A_4^i(0)B_4^j(0)]-[A_4^i(t)B_4^j(0)+A_4^i(0)B_4^j(t)]\cos(\omega t)\}-b_{ij}[A_4^i(t)B_4^j(0)-A_4^i(0)B_4^j(t)]\sin(\omega t)), \tag{C3}$$

$$J^{x}_{\pi_x\pi_x}(t)=\frac{2\hbar\omega_x\mu^3\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{A}_3^i(t)\dot{A}_3^j(t)+\dot{A}_3^i(0)\dot{A}_3^j(0)]-[\dot{A}_3^i(t)\dot{A}_3^j(0)+\dot{A}_3^i(0)\dot{A}_3^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_3^i(t)\dot{A}_3^j(0)-\dot{A}_3^i(0)\dot{A}_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{\pi_x\pi_x}(t)=\frac{2\hbar\omega_y\mu^3\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{B}_3^i(t)\dot{B}_3^j(t)+\dot{B}_3^i(0)\dot{B}_3^j(0)]-[\dot{B}_3^i(t)\dot{B}_3^j(0)+\dot{B}_3^i(0)\dot{B}_3^j(t)]\cos(\omega t)\}-b_{ij}[\dot{B}_3^i(t)\dot{B}_3^j(0)-\dot{B}_3^i(0)\dot{B}_3^j(t)]\sin(\omega t)), \tag{C4}$$

$$J^{x}_{\pi_y\pi_y}(t)=\frac{2\hbar\omega_x\mu^3\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{A}_4^i(t)\dot{A}_4^j(t)+\dot{A}_4^i(0)\dot{A}_4^j(0)]-[\dot{A}_4^i(t)\dot{A}_4^j(0)+\dot{A}_4^i(0)\dot{A}_4^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_4^i(t)\dot{A}_4^j(0)-\dot{A}_4^i(0)\dot{A}_4^j(t)]\sin(\omega t)),$$

$$J^{y}_{\pi_y\pi_y}(t)=\frac{2\hbar\omega_y\mu^3\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{B}_4^i(t)\dot{B}_4^j(t)+\dot{B}_4^i(0)\dot{B}_4^j(0)]-[\dot{B}_4^i(t)\dot{B}_4^j(0)+\dot{B}_4^i(0)\dot{B}_4^j(t)]\cos(\omega t)\}-b_{ij}[\dot{B}_4^i(t)\dot{B}_4^j(0)-\dot{B}_4^i(0)\dot{B}_4^j(t)]\sin(\omega t)), \tag{C5}$$

$$J^{x}_{\pi_x\pi_y}(t)=\frac{2\hbar\omega_x\mu^3\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{A}_3^i(t)\dot{B}_3^j(t)+\dot{A}_3^i(0)\dot{B}_3^j(0)]-[\dot{A}_3^i(t)\dot{B}_3^j(0)+\dot{A}_3^i(0)\dot{B}_3^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_3^i(t)\dot{B}_3^j(0)-\dot{A}_3^i(0)\dot{B}_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{\pi_x\pi_y}(t)=\frac{2\hbar\omega_y\mu^3\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{A}_4^i(t)\dot{B}_4^j(t)+\dot{A}_4^i(0)\dot{B}_4^j(0)]-[\dot{A}_4^i(t)\dot{B}_4^j(0)+\dot{A}_4^i(0)\dot{B}_4^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_4^i(t)\dot{B}_4^j(0)-\dot{A}_4^i(0)\dot{B}_4^j(t)]\sin(\omega t)), \tag{C6}$$

$$J^{x}_{x\pi_y}(t)=\frac{2\hbar\omega_x\mu^2\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[A_3^i(t)\dot{B}_3^j(t)+A_3^i(0)\dot{B}_3^j(0)]-[A_3^i(t)\dot{B}_3^j(0)+A_3^i(0)\dot{B}_3^j(t)]\cos(\omega t)\}-b_{ij}[A_3^i(t)\dot{B}_3^j(0)-A_3^i(0)\dot{B}_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{x\pi_y}(t)=\frac{2\hbar\omega_y\mu^2\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[A_4^i(t)\dot{B}_4^j(t)+A_4^i(0)\dot{B}_4^j(0)]-[A_4^i(t)\dot{B}_4^j(0)+A_4^i(0)\dot{B}_4^j(t)]\cos(\omega t)\}-b_{ij}[A_4^i(t)\dot{B}_4^j(0)-A_4^i(0)\dot{B}_4^j(t)]\sin(\omega t)), \tag{C7}$$

$$J^{x}_{y\pi_x}(t)=\frac{2\hbar\omega_x\mu^2\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}[a_{ij}\{[\dot{A}_3^i(t)B_3^j(t)+\dot{A}_3^i(0)B_3^j(0)]-[\dot{A}_3^i(t)B_3^j(0)+\dot{A}_3^i(0)B_3^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_3^i(t)B_3^j(0)-\dot{A}_3^i(0)B_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{y\pi_x}(t)=\frac{2\hbar\omega_y\mu^2\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}(a_{ij}\{[\dot{A}_4^i(t)B_4^j(t)+\dot{A}_4^i(0)B_4^j(0)]-[\dot{A}_4^i(t)B_4^j(0)+\dot{A}_4^i(0)B_4^j(t)]\cos(\omega t)\}-b_{ij}[\dot{A}_4^i(t)B_4^j(0)-\dot{A}_4^i(0)B_4^j(t)]\sin(\omega t)), \tag{C8}$$

$$J^{x}_{x\pi_x}(t)=\frac{2\hbar\omega_x\mu^2\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}[a_{ij}\{[A_3^i(t)\dot{A}_3^j(t)+A_3^i(0)\dot{A}_3^j(0)]-[A_3^i(t)\dot{A}_3^j(0)+A_3^i(0)\dot{A}_3^j(t)]\cos(\omega t)\}-b_{ij}[A_3^i(t)\dot{A}_3^j(0)-A_3^i(0)\dot{A}_3^j(t)]\sin(\omega t)),$$

$$J^{y}_{x\pi_x}(t)=\frac{2\hbar\omega_y\mu^2\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}[a_{ij}\{[B_3^i(t)\dot{B}_3^j(t)+B_3^i(0)\dot{B}_3^j(0)]-[B_3^i(t)\dot{B}_3^j(0)+B_3^i(0)\dot{B}_3^j(t)]\cos(\omega t)\}-b_{ij}[B_3^i(t)\dot{B}_3^j(0)-B_3^i(0)\dot{B}_3^j(t)]\sin(\omega t)), \tag{C9}$$

$$J^{x}_{y\pi_y}(t)=\frac{2\hbar\omega_x\mu^2\lambda_x^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}[a_{ij}\{[A_4^i(t)\dot{A}_4^j(t)+A_4^i(0)\dot{A}_4^j(0)]-[A_4^i(t)\dot{A}_4^j(0)+A_4^i(0)\dot{A}_4^j(t)]\cos(\omega t)\}-b_{ij}[A_4^i(t)\dot{A}_4^j(0)-A_4^i(0)\dot{A}_4^j(t)]\sin(\omega t)),$$

$$J^{y}_{y\pi_y}(t) = \frac{2\hbar\omega_y\mu^2\lambda_y^0\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega\frac{\omega[2n_\omega+1]}{\gamma^2+\omega^2}[a_{ij}\{[B_4^i(t)\dot{B}_4^j(t) + B_4^i(0)\dot{B}_4^j(0)] - [B_4^i(t)\dot{B}_4^j(0) + B_4^i(0)\dot{B}_4^j(t)]\cos(\omega t)\} - b_{ij}[B_4^i(t)\dot{B}_4^j(0) - B_4^i(0)\dot{B}_4^j(t)]\sin(\omega t)). \quad (C10)$$

Here, $A_k(t)=\Sigma_{i=1}^6 A_k^i(t)$, $B_k(t)=\Sigma_{i=1}^6 B_k^i(t)$, $k=1-4$, and

$$a_{ij} = \frac{s_i s_j + \omega^2}{(s_i^2+\omega^2)(s_j^2+\omega^2)},$$

$$b_{ij} = \frac{(s_j - s_i)\omega}{(s_i^2+\omega^2)(s_j^2+\omega^2)}.$$

The asymptotic variances are

$$\sigma_{xx}(\infty) = J_{xx}(\infty) = \frac{2\hbar\gamma^2}{\pi\mu}\int_0^\infty d\omega\omega[2n_\omega+1]\frac{\omega_x\lambda_x^0[\gamma^2(\omega^2-\omega_y^2+2\gamma\omega_y\lambda_y^0)^2+\omega^2(\omega^2-\omega_y^2)^2]+\omega_L^2\omega_y\lambda_y^0\omega^2(\omega^2+\gamma^2)}{(s_1^2+\omega^2)(s_2^2+\omega^2)(s_3^2+\omega^2)(s_4^2+\omega^2)(s_5^2+\omega^2)(s_6^2+\omega^2)}, \quad (C11)$$

$$\sigma_{yy}(\infty) = J_{yy}(\infty) = \frac{2\hbar\gamma^2}{\pi\mu}\int_0^\infty d\omega\omega[2n_\omega+1]\frac{\omega_y\lambda_y^0[\gamma^2(\omega^2-\omega_x^2+2\gamma\omega_x\lambda_x^0)^2+\omega^2(\omega^2-\omega_x^2)^2]+\omega_L^2\omega_x\lambda_x^0\omega^2(\omega^2+\gamma^2)}{(s_1^2+\omega^2)(s_2^2+\omega^2)(s_3^2+\omega^2)(s_4^2+\omega^2)(s_5^2+\omega^2)(s_6^2+\omega^2)}, \quad (C12)$$

$$\sigma_{\pi_x\pi_x}(\infty) = J_{\pi_x\pi_x}(\infty) = \frac{2\mu\hbar\gamma^2}{\pi}\int_0^\infty d\omega\omega^3[2n_\omega+1]\frac{\omega_x\lambda_x^0[\gamma^2(\omega^2-\omega_y^2+2\gamma\omega_y\lambda_y^0)^2+\omega^2(\omega^2-\omega_y^2)^2]+\omega_L^2\omega_y\lambda_y^0\omega^2(\omega^2+\gamma^2)}{(s_1^2+\omega^2)(s_2^2+\omega^2)(s_3^2+\omega^2)(s_4^2+\omega^2)(s_5^2+\omega^2)(s_6^2+\omega^2)}, \quad (C13)$$

$$\sigma_{\pi_y\pi_y}(\infty) = J_{\pi_y\pi_y}(\infty) = \frac{2\mu\hbar\gamma^2}{\pi}\int_0^\infty d\omega\omega^3[2n_\omega+1]\frac{\omega_y\lambda_y^0[\gamma^2(\omega^2-\omega_x^2+2\gamma\omega_x\lambda_x^0)^2+\omega^2(\omega^2-\omega_x^2)^2]+\omega_L^2\omega_x\lambda_x^0\omega^2(\omega^2+\gamma^2)}{(s_1^2+\omega^2)(s_2^2+\omega^2)(s_3^2+\omega^2)(s_4^2+\omega^2)(s_5^2+\omega^2)(s_6^2+\omega^2)}, \quad (C14)$$

$$\sigma_{x\pi_y}(\infty) = J_{x\pi_y}(\infty) = -\sigma_{y\pi_x}(\infty) = -J_{y\pi_x}(\infty) = -\frac{2\hbar\omega_L\gamma^2}{\pi}\int_0^\infty d\omega\omega^3[2n_\omega+1] \times \frac{\omega_x\lambda_x^0[(\omega^2+\gamma^2)(\omega^2-\omega_y^2)+2\omega_y\lambda_y^0\gamma^3]+\omega_y\lambda_y^0[(\omega^2+\gamma^2)(\omega^2-\omega_x^2)+2\omega_x\lambda_x^0\gamma^3]}{(s_1^2+\omega^2)(s_2^2+\omega^2)(s_3^2+\omega^2)(s_4^2+\omega^2)(s_5^2+\omega^2)(s_6^2+\omega^2)}. \quad (C15)$$

Here, $2n_\omega+1=\coth[\hbar\omega/(2T)]$. The asymptotic variances $\sigma_{\pi_x\pi_y}(\infty)=J_{\pi_x\pi_y}(\infty)$, $\sigma_{x\pi_x}(\infty)=J_{x\pi_x}(\infty)$, $\sigma_{y\pi_y}(\infty)=J_{y\pi_y}(\infty)$, and $\sigma_{xy}(\infty)=J_{xy}(\infty)$ are equal to zero. It is seen from Eq. (C15) that $\sigma_{x\pi_y}=\sigma_{y\pi_x}=0$ when $\omega_L=0$.

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